Remarks on an Apparatus for determining those Errors of astronomical Observations which are caused by the Flexures of an Instrument and by Defects in the Shape of its Pivots. By A. Marth, Esq.

At the meeting of last December, in the course of the discussion on Mr. Stone's paper on a new form of Transit-Circle, I stated that in the spring of 1862 I had published* a proposal of a new method for determining the flexure- and pivot-errors, which was specially applicable in the case of an instrument with a prism in the centre—a case to which Mr. Stone had alluded. I mentioned that in 1869 an apparatus such as I proposed had been constructed by Messrs. T. Cooke & Sons, of York, for a Transit-Circle with telescope of the ordinary form, and that the instrument was still at their works. I further remarked that, in December 1878, there had appeared in the "Comptes Rendus" of the Paris Academy a paper by M. Loewy, in which he raised some objections to my proposal which had been a great puzzle to me; that, while in his own proposal M. Loewy had adopted the principle of my method, he had made alterations in its application about which astronomers would form their own judgment by-and-by. I mentioned also that Messrs. Cooke would probably bring or send up their apparatus for inspection at one of our meetings, so that those who take an interest in the matter—and especially Mr. Stone—might criticise it and compare it with M. Loewy's, of which a description had lately appeared.

The apparatus is to be shown at the May meeting, and I will, therefore, now offer some explanatory remarks. But before doing so, it will be desirable or even essential, on account of some extraordinary assertions which M. Loewy makes in the April number of the *Monthly Notices*, that I should first submit to the critical consideration of readers a preliminary question referring to elementary geometrical optics.

If a distant object is looked at through a refracting telescope, how is the place of the object's image in the field of view affected by some slight shifting of the object-glass?

The question must have presented itself to every thinking observer who uses an astronomical instrument intelligently, so that there should be no difficulty in getting an answer, even if the text-books are silent about it. I have not been aware that there was or could be a doubt about the correctness of the answer, that the effect of the shifting depends entirely upon the shifting of the *optical centre* of the object-glass (if by optical centre is understood that point where the straight lines cross which join the points of an infinitely distant object with the

* *Astr. Nachrichten*, No. 1361 in Vol. LIII. "Vorschlag eines neuen Verfahrens, die von der Biegung eines Instruments und von den Unregelmässigkeiten seiner Zapfen erzeugten astronomischen Beobachtungsfehler zu bestimmen."

corresponding points of its image), and that any slight shifting of the object-glass round its optical centre does *not* affect the line of vision or the direction in which the object appears. I cannot find, at present, any flaw in the reasoning which leads to this conclusion. But if the answer is really wrong, I shall be much obliged to any reader who will enlighten me (and probably others) about what is the truth in the matter, and I shall *then* most readily acknowledge that I have been all along under a delusion respecting an elementary question of practical optics.

The exact position of the optical centre and other cardinal points in any system of lenses on a common axis may be determined either from the optical elements of the system, in case these are known, or by means of special experiments, the theory having been cleared up by the dioptrical investigations of Gauss * and Bessel † published nearly simultaneously at the beginning of 1841.

As some readers may be interested to get distinct notions of the actual forms of two celebrated object-glasses, of which the optical elements are known, they will perhaps avail themselves of the following data for the graphical representation of sections of these glasses passing through the axis of the lenses. The x abscissæ, expressed in English inches, are reckoned along the axis from the centre of the outer surface of the crown lens, the y ordinates perpendicular to the axis.

Object-glass of the Koenigsberg Heliometer.

The elements of its construction are given on p. 101 of Bessel's "Astr. Untersuchungen."

	$y=0$	± 1·07	± 2·13	± 3·20
Crown lens	$x=0·0$	·008	·031	·069
	$=0·234$	First principal point.		
	$=0·533$	·514	·456	·360
Flint lens	$=0·533$	·514	·458	·363
	$=0·556$	Second principal point = optical centre.		
	$=0·888$	·883	·866	·839

Focal length 100in·50, effective aperture 6in·23.

Object-glass of the great Washington Equatoreal.

According to Prof. Holden's "Investigation of the Objective and Micrometers of the Twenty-six Inch Equatoreal" (Appendix

* Gauss, "Dioptrische Untersuchungen," *Werke*, Bd. V. The optical centre in the sense of the term as used above, is identical with the point called by Gauss "zweiter Hauptpunkt."

† Bessel, "Ueber die Grundformeln der Dioptrik," *Astr. Nachr.* No. 415. Cf. in the *Briefwechsel zwischen Gauss und Bessel* the letters No. 179 and 180, exchanged in January 1841. The further investigations of Bessel must be studied in the "Besondere Untersuchung des Heliometers . . . . ," the second paper of his *Astron. Untersuchungen.*

I. of Wash. Obs. for 1877), the elements of its construction are approximately (using the notation employed by Bessel)—

$$\left.\begin{array}{l} r = +\ 161{\cdot}39 \text{ in.} \\ d = 1{\cdot}884 \\ \rho = -\ 161{\cdot}39 \end{array}\right\} n = 1{\cdot}5172$$

$$e = 0{\cdot}029$$

$$\left.\begin{array}{l} r' = -\ 162{\cdot}07 \\ d' = 0{\cdot}958 \\ \rho' = -\ 19466{\cdot} \end{array}\right\} n' = 1{\cdot}6280$$

Hence the coordinates for graphical representation of a section of the object-glass:

	$y=$	0	±3	±6	±9	±12	±13	±13·6
	$x=$	−0·344	First principal point.					
Crown lens	$=$	0·0	0·028	0·112	0·251	0·447	0·524	0·574
	$=$	0·669	Second principal point = optical centre.					
	$=$	1·884	1·856	1·772	1·633	1·437	1·360	1·310
Flint lens	$=$	1·913	1·885	1·802	1·663	1·468	1·391	1·341
	$=$	2·871	2·871	2·870	2·869	2·867	2·867	2·866

The focal length, deduced from the assumed elements by means of the catenary fraction employed by Bessel, is 389·12 inches; Prof. Holden, using a modification of Gauss's formulæ, finds it = 389·66 inches.

While the optical centre of the object-glass of the Koenigsberg Heliometer is a little within the flint lens very near to the point where the two lenses touch, that of the great Washington object-glass is within the crown lens, at about one-third of its thickness from the outer surface.

With the exception of a few cases, the elements of the construction of the object-glasses employed are not known to observers. The exact places of the optical centres cannot, on that account, be computed, and their practical determination is only feasible where the means are available for making proper experiments for the purpose. For ordinary observations a knowledge of the position of the optical centre may, indeed, be dispensed with; it may be sufficient to know that there is such a point through which the line of vision passes. In applying my method for determining flexure, in which a marked point on the object-glass is observed, it will be desirable to know the place of the optical centre at least approximately, so that the effect of any shifting of the object-glass may be duly estimated.

I will now proceed to explain how my old proposal of an apparatus for determining the astronomical errors which affect the line of vision has been carried out in Messrs. Cooke's

apparatus which is to be shown at the meeting. The Transit-Circle to which the apparatus belongs has an object-glass of 7 inches aperture and 88 inches focal length. The pivots of the axis have apertures of $2\frac{1}{4}$ inches diameter in order to allow the determination of the errors arising from flexure of the axis and shape of the pivots. The axis of the instrument is cast in one piece; the middle part is 13 inches across and has in the middle of its two exposed sides apertures in the form of squares of 4 inches, with rounded-off corners. Usually these apertures are closed by covers, which are held in their places by press-clamps attached to the instrument. When the flexure-apparatus is to be used, these covers (which together have the same weight as the apparatus, about 14 lbs.) are removed and the apparatus is inserted so that its stout end-pieces or end-plates fill the apertures, and are held there so that the clamped parts are flush with the sides of the instrument. The stout end-plates fit very closely, but some small portions of the metal are to be taken off on two sides of the apparatus in order to allow the insertion of wedges so as to guard against any possibility of shake. The connection of the apparatus with the central part of the instrument is very strong and yet very simple; in planning this connection it was made a condition that the apparatus should be fit to be placed in the right position without troublesome preparations, so that it might be ready for use at any spare time in the regular course of observations. The middle part of the apparatus contains a plane-parallel glass between two achromatic object-glasses of $2\frac{1}{2}$ inches aperture and of nearly half the focal length of the chief object-glass. One half of the plane glass is to be silvered on both sides, so that the silvered half may serve as a mirror. The silvered portion may be the middle or the outer part; or a half-circle or the whole may be silvered, according to the observer's judgment. The frame which holds the mirror may be taken out and inserted in the reversed position; of course, proper care must be taken to fasten it so as to guard against any possibility of shake. The two small object-glasses must be so placed that the mark on the great object-glass is in the focal plane of the one, while the focus of the other is in the plane of the webs at the eye-end. In order to enable the observer to effect exact focussing, the cells of the small object-glasses can be slowly moved in the direction of their axes by being turned round with the help of wheel-work, the last wheels of which are outside the end-plates of the apparatus, within the handles, where they may be turned from the eye-end. After focussing, the cells may be further fastened, if need be; but, as an inspection of the apparatus will show, Messrs. Cooke's contrivance is so well executed that the precaution will probably be found superfluous.

When the apparatus and the illumination are got ready, there are to be seen in the field of view: the webs, their image reflected by the mirror, and the image of the cross-lines by

which the chief object-glass is marked. The relative positions of these images (in the direction of the transit-observations as well as of the circle-observations) are then to be measured in different directions of the telescope. The measured changes in the relative positions of the images are the effect of the flexure of the telescope, yet mixed up with some possible changes in the position of the mirror relative to the parts of the instrument where the apparatus is fastened, and also with some possible changes in the relative position of the optical centres of the two small object-glasses. Moreover, the line of vision is thus referred to the actual axis of rotation of the apparatus, the direction of which is liable to change on account of flexure of the axis of the instrument, and of possible irregularities in the pivots and their supports. For the determination of the united effect of these latter sources of error, the flexure apparatus carries at the side facing one of the pivots a plane mirror. Further, on an entirely independent support, outside of the instrument, an auxiliary telescope with micrometer, &c., is to be firmly mounted, so that it points through the aperture in the pivot of the Transit-Circle to the side-mirror of the flexure-apparatus. By bringing the (horizontal and vertical) webs of the auxiliary micrometer to coincidence with their image reflected from the side mirror, and making the observation for many readings of the Transit-Circle, the coordinates of the curve become known which the normal to the mirror describes on the sphere, and from them the effect of the flexure of the axis and of the pivot-errors upon the line of vision of the Transit-Circle telescope is found.

In order to examine and to eliminate the effects of any possible changes in the position of the central mirror, &c., and of other possible sources of error, the flexure-apparatus is to be reversed as well in regard to the axis of the telescope as in regard to the axis of rotation, and the observations are to be made in the four positions of the apparatus. I will not dilate, at present, further on this question. As my remarks are intended for critical readers who are capable of forming a judgment of their own, and who are sufficiently interested in the subject not to mind the trouble of looking over my old paper of 1862, I should be obliged for their critical examination of what I have stated there on this point, so that we may get at the truth. They will perhaps also form their own judgment on the curious objections and assertions which M. Loewy makes in his paper in the last number of the *Monthly Notices*, and on which I shall have something to say in the next number.

At the December meeting I stated that my old proposal was especially applicable in the case of an instrument with a prism in the centre, as, in fact, I had mentioned long ago in the introductory remarks of my old paper. The advantages of such an instrument are obvious: one of the most important, where the highest attainable accuracy is required, is the absence of a very serious source of error—namely, the presence of the observer's

body underneath or close to the instrument. The question whether it is preferable to break the line of vision by a prism or by a piece of silvered glass will be settled by considerations of the available facilities for renewing the silvered surface. If a prism is preferred, I may perhaps mention an old suggestion of mine to make use of an elongated prism, of which only the middle part need be brought to perfection for reflection, while the ends are to serve for the firmer mounting of the prism. But I will not enter here into considerations of the proper construction of the middle part of the axis of the instrument for securing strength and the firm mounting of the prism and of the flexure-apparatus. I will merely indicate the simplified conditions which the latter has to fulfil. The side-mirror, of course, disappears. As the apparatus is mounted near the prism at the side of the eye-end, the lenses and the mirror turn, with the telescope, round their own axes, and reversion with regard to the line of vision becomes superfluous. The small object-glasses, the apertures of which ought not to be less than the side of the prism, are now of unequal focal length, since, for the sake of using a prism of moderate size, the distance to the webs in the eye-end will probably be made considerably shorter than that to the mark on the great object-glass. The flexure-apparatus will perhaps be best so arranged that both the small object-glasses may be taken out and the (fully) silvered mirror put in place of that farthest from the eye-end. When the eye-end micrometer is removed and the auxiliary telescope employed, the curve described by the normal to the plane of the mirror is referred to a fixed direction. If, then, the small object-glass nearest the eye-end is inserted, and the eye-end micrometer reinserted and adjusted, the observed coincidences of the webs with their reflected image will refer the positions of the webs to the normal of the mirror. If, further, the mirror is replaced by the other object-glass, the changes of the image of the mark in reference to a zero point of the net of webs will show the effect of the flexure of the telescope-tube and of the bending of the prism, this effect multiplied by the proportion of the focal lengths of the two object-glasses and added to the changes of the zero point. These experiments may then be varied, &c. But the hints here given will perhaps be sufficient for thinking readers who take an interest in the problem, to induce them to consider it in all its bearings, and I must leave it to those astronomers who have means and opportunities to get ideas carried out in practice, whether they will avail themselves of these suggestions.